

$B \rightarrow X_s + \text{Missing Energy}$ in unparticle model

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Abstract

We analyze the inclusive decay mode $B \rightarrow X_s + \text{Missing Energy}$ in the unparticle model, where an unparticle can also serve as the missing energy. We use the Heavy Quark Effective Theory in the calculation. The analytical result of the decay width in the free quark limit and that of the differential decay rate to the order of $1/m_b^2$ are presented. Numerical results of the inclusive mode show interesting differences from those of the exclusive modes. Near the lower endpoint region, the $d_{\mathcal{U}} < 2$ unparticle has very different behavior from the Standard Model particles.

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1 Introduction

The rare decay $B \rightarrow X_s + \nu\bar{\nu}$ is very small in the Standard Model (SM)[1] so that it might be very sensitive to the new physics beyond the SM. Furthermore, in the final state the missing energy carried by the neutrino-anti-neutrino pair might be polluted experimentally by other missing-energy-like states in the presence of new physics.

Recently, the Unparticle Model suggested by Georgi[2] provides a possible candidate for the missing energy in $B \rightarrow X_s + \text{Missing Energy}$. In the Unparticle Model, below a scale $\Lambda_{\mathcal{U}}$ the interaction of the unparticle with the SM sector takes a form like[2]

$$\frac{\mathcal{C}_{\mathcal{U}}\Lambda_{\mathcal{U}}^{d_{\mathcal{B}\mathcal{Z}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}\mathcal{O}_{SM}\mathcal{O}_{\mathcal{U}}, \quad (1)$$

where $\mathcal{C}_{\mathcal{U}}$ is a coefficient function and $M_{\mathcal{U}}$ is a large mass scale of the particles mediating the interaction between the SM fields and the unparticle fields. If $M_{\mathcal{U}}$ is large enough, the unparticle stuff does not couple strongly to the ordinary particles. People have introduced many kinds of couplings[2, 3, 4], including the Yukawa and the partial differential couplings. A lot of works have been done to study the possible consequence of the unparticle, most of which focus on the exclusive processes. Here we will discuss the inclusive mode $B \rightarrow X_s + \text{Missing Energy}$, treating the unparticle as part of the missing energy. We will use Heavy Quark Effective Theory (HQET) in our analysis. The results are constrained by the data [5].

The organization of this paper is as follows. At first, we apply the HQET results to unparticle model and give out the general form of $B \rightarrow X_s + \mathcal{U}$ decay rates. Then we present in the analytical forms the decay width in the free quark approximation and the differential decay width versus the missing energy to the $1/m_b^2$ order. Numerical results will be given. We will summarize at the end.

2 HQET application in Unparticle Model

In the Unparticle Model, the effective Hamiltonian for $B \rightarrow X_s + \mathcal{U}$ at quark level is given by

$$\mathcal{H}_{eff}^{\mathcal{S}} = \frac{\mathcal{C}_{\mathcal{S}}^q\Lambda_{\mathcal{U}}^{k-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}(\bar{b}\gamma_{\mu}(1-\gamma_5)s)\partial^{\mu}\mathcal{O}_{\mathcal{U}} \quad (2)$$

for the scalar unparticle, or

$$\mathcal{H}_{eff}^{\mathcal{V}} = \frac{\mathcal{C}_{\mathcal{V}}^q\Lambda_{\mathcal{U}}^{k+1-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}(\bar{b}\gamma_{\mu}(1-\gamma_5)s)\mathcal{O}_{\mathcal{U}}^{\mu} \quad (3)$$

for the vector unparticle. Here $\mathcal{C}_{\mathcal{S}}^q$ and $\mathcal{C}_{\mathcal{V}}^q$ are the dimensionless coupling constant between the quark current and unparticle fields. The $d_{\mathcal{U}}$ is a non-integral number severing as the dimension of unparticle operators. The $d_{\mathcal{U}}$ can not be small than 1 for the unitary of the theory[2]. If one impose the conformal symmetry on the vector unparticle fields, the primary, gauge invariant vector unparticle operators could only have dimension $d_{\mathcal{U}} > 3$ [6, 7].

We will also introduce the two dimensional coefficients corresponding to scalar and vector unparticles

$$c_S^q = \frac{\mathcal{C}_S^q \Lambda_{\mathcal{U}}^{k-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}, \quad \text{and} \quad c_V^q = \frac{\mathcal{C}_V^q \Lambda_{\mathcal{U}}^{k+1-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}. \quad (4)$$

The most general forms of differential decay rates for the scalar and vector unparticle final states are

$$\begin{aligned} d\Gamma^{S(V)} &= \sum_{X_s} (2\pi)^4 \delta^4(p_B - p_{\mathcal{U}} - p_{X_s}) \frac{|\langle X_s \mathcal{U} | H_{eff}^{S(V)} | B \rangle|^2}{2m_B} \\ &\times A_{d_{\mathcal{U}}} \theta(p_{\mathcal{U}}^0) \theta(p_{\mathcal{U}}^2) (p_{\mathcal{U}}^2)^{d_{\mathcal{U}}-2} \frac{d^4 p_{\mathcal{U}}}{(2\pi)^4}, \end{aligned} \quad (5)$$

where $A_{d_{\mathcal{U}}}$ is defined as[2]

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + 1/2)}{\Gamma(d_{\mathcal{U}} - 1)\Gamma(2d_{\mathcal{U}})}. \quad (6)$$

The factor $A_{d_{\mathcal{U}}} \theta(p_{\mathcal{U}}^0) \theta(p_{\mathcal{U}}^2) (p_{\mathcal{U}}^2)^{d_{\mathcal{U}}-2}$ in (5) counts for the unparticle phase space[2], over which the integrations can be performed in the rest frame of the B meson. The rest part in (5) containing the matrix element squared are conventionally written, in analogy with those in the semileptonic decays in the SM, as the product of the hadron and the unparticle tensors analytical [8, 9, 10, 11, 12],

$$\begin{aligned} &\sum_{X_s} (2\pi)^3 \delta^4(p_B - p_{\mathcal{U}} - p_{X_s}) \frac{|\langle X_s \mathcal{U} | H_{eff}^{S(V)} | B \rangle|^2}{2m_B} \\ &= c_{S(V)}^{q^2} W_{\alpha\beta} U_{S(V)}^{\alpha\beta}, \end{aligned} \quad (7)$$

where the unparticle tensors are

$$U_S^{\alpha\beta} = p_{\mathcal{U}}^\alpha p_{\mathcal{U}}^\beta \quad \text{and} \quad U_V^{\alpha\beta} = -g^{\alpha\beta} + \frac{p_{\mathcal{U}}^\alpha p_{\mathcal{U}}^\beta}{p_{\mathcal{U}}^2}, \quad (8)$$

and the hadronic tensor is defined by

$$W_{\alpha\beta} = \sum_{X_s} (2\pi)^3 \delta^4(p_B - q - p_{X_s}) \frac{\langle B(p_B) | J_\alpha^\dagger | X_s(p_{X_s}) \rangle \langle X_s(p_{X_s}) | J_\beta | B(p_B) \rangle}{2m_B}, \quad (9)$$

with $J_\alpha = \bar{s} \gamma_\alpha (\frac{1-\gamma_5}{2}) b$.

The most general form of $W_{\alpha\beta}$ is[9]

$$W_{\alpha\beta} = -g_{\alpha\beta} W_1 + v_\alpha v_\beta W_2 - i\epsilon_{\alpha\beta\gamma\delta} v^\gamma q^\delta W_3 + q_\alpha q_\beta W_4 + (v_\alpha q_\beta + q_\alpha v_\beta) W_5. \quad (10)$$

The scalar structure functions W_j are functions of q^2 and $v \cdot q$, where v is the four velocity of the heavy bottom quark. They are related to T_j 's by applying $W_j = -1/\pi \text{Im} T_j$, where

$$T_{\alpha\beta} = -i \int d^4 x e^{-iq \cdot x} \frac{\langle B | T[J_\alpha^\dagger(x) J_\beta(0)] | B \rangle}{2m_B}. \quad (11)$$

In general,

$$T_{\alpha\beta} = -g_{\alpha\beta}T_1 + v_\alpha v_\beta T_2 - i\epsilon_{\alpha\beta\gamma\delta}v^\gamma q^\delta T_3 + q_\alpha q_\beta T_4 + (v_\alpha q_\beta + q_\alpha v_\beta)T_5. \quad (12)$$

The HQET provides a systematical tool in investigating the Heavy-light hadrons such like the B mesons[8, 9]. T_j 's can be expanded in $1/m_b$ using HQET, their forms to $1/m_b^2$ can be found in Ref.[11, 12]. There emerge some problems near the endpoint region of the energy spectrum[10], which can be avoided by introducing suitable cuts in this work.

Applying Eqs.(7,8,10) we get

$$\begin{aligned} d\Gamma^S &= 8\pi c_S^{q^2} [-p_U^2 W_1 + (v \cdot p_U)^2 W_2 + (p_U^2)^2 W_4 + 2(v \cdot p_U) p_U^2 W_5] \\ &\times A_{d_U} \theta(p_U^0) \theta(p_U^2) (p_U^2)^{d_U-2} \frac{d^4 p_U}{(2\pi)^4}, \end{aligned} \quad (13)$$

and

$$d\Gamma^V = 8\pi c_V^{q^2} \left[-3W_1 + \left(1 - \frac{(v \cdot p_U)^2}{p_U^2}\right) W_2 \right] A_{d_U} \theta(p_U^0) \theta(p_U^2) (p_U^2)^{d_U-2} \frac{d^4 p_U}{(2\pi)^4}. \quad (14)$$

The integrations over $d^4 p_U$ in (13) and (14) are constrained by the function $\theta(p_U^0)\theta(p_U^2)$ and by the condition $p_U^0 < m_B - m_{X_{smin}}$.

3 Differential decay rates and the width

The inclusive differential decay rates are calculated using (13) and (14) by taking $m_s \rightarrow 0$. Terms with derivatives of δ function are evaluated using integrating by parts. Then the differential decay width for the scalar unparticle emission is

$$\begin{aligned} \frac{d\Gamma^S}{dx} &= \frac{A_{d_U} c_S^{q^2} m_b^{2d_U-1} (2x-1)^{d_U-4}}{3\pi^2 (x-1)} \left\{ \theta(x-1/2) \left[3/2 m_b^2 (2x-1)^2 (x-1)^3 \right. \right. \\ &+ 1/2 \lambda_1 \left(2d_U^2 (8x^2 - 5x + 1)(x-1)^4 - d_U (48x^4 + 40x^3 - 97x^2 + 47x - 6)(x-1)^2 \right. \\ &+ 50x^4 + 3 - 18x - 40x^5 + 49x^2 + 32x^6 - 66x^3 \Big) \\ &- 3/4 \lambda_2 (2x-1) \left(2d_U (37x^2 - 39x + 10)(x-1)^2 - 20x^4 - 19x - 8x^3 + 36x^2 + 3 \right) \Big] \\ &+ 1/4 \delta(x-1/2) (2x-1)(x-1)^2 \left[\lambda_1 \left((4(8x^2 - 5x + 1)(x-1)^2 d_U \right. \right. \\ &+ 8 + 135x^2 - 82x^3 - 61x - 32x^4) \Big) + 3\lambda_2 (2x-1)(37x^2 - 39x + 10) \Big] \\ &\left. \left. + 1/4 \delta'(x-1/2) \lambda_1 (8x^2 - 5x + 1)(2x-1)^2 (x-1)^4 \right\}, \end{aligned} \quad (15)$$

and for the vector unparticle emission, it is

$$\begin{aligned}
\frac{d\Gamma^\nu}{dx} = & \frac{A_{d_U} c_V^{q^2} m_b^{2d_U-3} (2x-1)^{d_U-5}}{12\pi^2(x-1)} \left\{ \theta(x-1/2) \left[-6m_b^2(3-9x+8x^2)(2x-1)^2(x-1)^2 \right. \right. \\
& + \lambda_1 \left(4d_U^2(3-9x+8x^2)(x-1)^4 + 2d_U(12x^3-19x^2+15x-6)(x-1)^2 \right. \\
& \left. \left. -12x-128x^6-306x^4+6+316x^5-14x^2+130x^3 \right) \right. \\
& + \lambda_2 \left(6d_U(2x-1)(40x^3-85x^2+69x-18)(x-1)^2 \right. \\
& \left. \left. -3(2x-1)(160x^5-518x^4+722x^3-544x^2+225x-39) \right) \right] \\
& + \delta(x-1/2)(x-1)^2(2x-1) \left[\lambda_1 \left(4d_U(3-9x+8x^2)(x-1)^2 \right. \right. \\
& \left. \left. + x(39x-15+16x^3-38x^2) \right) + \lambda_2(6x-3)(40x^3-85x^2+69x-18) \right] \\
& \left. + \delta'(x-1/2)\lambda_1(x-1)^4(2x-1)^2(3-9x+8x^2) \right\}, \tag{16}
\end{aligned}$$

where λ_1 and λ_2 are parameters in the heavy quark expansion. Here we introduce the dimensionless variable $x = p_U^0/m_b$, which serves as unparticle energy modulated by the heavy quark mass.

Further integration over x will not generally give analytical results for an arbitrary d_U because of the factor $(2x-1)^{2d_U-4(\text{or } 5)}/(x-1)$ in Eq.15(or 16). However, when taking the limit $\lambda_1 = \lambda_2 = 0$ we can integrate over x . We get

$$\Gamma^S = A_{d_U} \frac{c_S^{q^2}}{8\pi^2} \frac{m_b^{2d_U+1}}{(d_U^2-1)d_U}, \tag{17}$$

and

$$\Gamma^\nu = \frac{c_V^{q^2} A_{d_U} m_b^{2d_U-1} (2d_U^2 - 5d_U + 5)}{8\pi^2 d_U (d_U - 1)(d_U + 1)(d_U - 2)} \quad \text{for } d_U > 2. \tag{18}$$

Eqs.(17) and (18) are the decay widths for $b \rightarrow s + \mathcal{U}$ in the free quark approximation.

We find that, even if $d_U = 1$, the scalar unparticle in the final state gives a finite contribution in (17), as a result of the fact that the singularity in the factor $(d_U-1)^{-1}$ is compensated by the factor A_{d_U} . This is not a common feature as in the exclusive processes such like in Ref.[13]. We cannot give a simple analytical formula like (18) for the vector unparticle in the final state when $d_U < 2$. But under the assumption of exact conform symmetry d_U must be bigger than 3[6], then Eq.(18) is enough at the first order.

4 Numerical results

Numerical results are needed in order to exhibit the unparticle effects. Both the unparticle and neutrino-anti-neutrino pairs serve as the missing energy \cancel{E} . We can not distinguish them

in the experiments, so the process $B \rightarrow X_s + \cancel{E}$ may contain both $\nu\bar{\nu}$ and \mathcal{U} in the final states. The decay width is

$$\Gamma(B \rightarrow X_s + \cancel{E}) = \Gamma(B \rightarrow X_s \nu\bar{\nu}) + \Gamma(B \rightarrow X_s \mathcal{U}), \quad (19)$$

where $\Gamma(B \rightarrow X_s \nu\bar{\nu})$ comes from the SM and $\Gamma(B \rightarrow X_s \mathcal{U})$ come from either the scalar or the vector unparticle contribution.

Present calculation in SM[1, 14] gives

$$B_{SM}(B \rightarrow X_s + \nu\bar{\nu}) = (3.4 \pm 0.7) \times 10^{-5}. \quad (20)$$

The experimental bound[5],

$$B_{exp}(B \rightarrow X_s + \cancel{E}) < 6.4 \times 10^{-4}, \quad (21)$$

is about one order larger than the SM calculation. This large difference allows new physics to provide candidates as the missing energy. In the unparticle model the candidate is the unparticle.

There are two kinds of singularities brought by the unparticle: one comes from the $(2x-1)^{d_U-4(5)}$ and the other from $1/(x-1)$. At the quark level, the endpoint of unparticle energy spectrum is singular. But the true endpoint are at $(m_B^2 - m_K^2)/2m_B$ and $m_B - m_K$, depending on the masses of the hadrons. Near $m_B - m_K$ or $x \sim 1$ when the HQET fails, we introduce some cuts. For the scalar unparticle model, we set the $x = 0.975$, *i. e.* we take the $\Lambda_{QCD} \sim 200\text{MeV}$ that around $m_B - \Lambda_{QCD}$ the HQET fails[10]. But for the vector unparticle theory, the case is different because the $\theta(1-x)$ term in (16) goes through zero when $x \sim 0.923$, so it serves as our cut. We have taken the mass parameters as[5, 15]

$$m_b = 4.7\text{GeV}, m_B = 5.279\text{GeV}, \text{ and } m_K = 0.493\text{GeV}, \quad (22)$$

and the heavy quark expansion parameters as [16]

$$\lambda_1 = -0.497\text{GeV}^2 \text{ and } \lambda_2 = 0.12\text{GeV}^2. \quad (23)$$

There are very distinctive differences between the unparticle model and the SM in the differential decay widths when $d_U < 2$. Near the lower endpoint region $x \sim (m_B^2 - m_K^2)/2m_B m_b$, the SM differential width approach zero, while the unparticle model gives finite results. This comes from $(p_U^2)^{d_U-2}$ in the phase space of the unparticle, see Eqs.(13) and (14). When p_U^2 goes to zero, the phase space goes to infinity. But the SM final state $\nu\bar{\nu}$ has no such an enhancement. For the vector unparticle model, the differential width also gives a finite result at $x \sim (m_B^2 - m_K^2)/2m_B m_b$ when $d_U > 2$. Here, the enhancement comes from the vector unparticle tensor $U_V^{\alpha\beta}$ in Eq.(8). p_U^2 appears in the denominator.

Near the endpoint $x \sim (m_B - m_K)/m_b$, the scalar unparticle model results go to infinity, which comes from the HQET. There are also such kinds of singularity in SM[9, 18, 19]. For the vector unparticle model, the results go to negative infinity when x goes to 1. And the vector unparticle contributions vanish near $x = 1$. This turning comes from the form of matrix element in (14) and HQET.

We plot the spectra for the scalar unparticle model in Fig. 1 for $d_U \leq 2$ and Fig. 2 for $d_U > 2$, respectively. The spectra for vector unparticle model are given in Fig. 3. for $d_U < 3$, and in Fig. 4 for $d_U \geq 3$, respectively. Note that $d_U < 3$ is allowed if we give up the strict conformal symmetry.

The branching ratios of scalar and vector unparticle emission processes show many resemblances if one neglect the conformal constraint on d_U . They are presented in Fig. 5 for the scalar unparticle, and in Fig. 6 and 7 for the vector unparticle. Figs. 6 is allowed if one gives up the strict conformal symmetry, with the unparticle contribution peaking at $d_U = 1.3 \sim 1.4$. In this case the branching ratio goes down when d_U becomes bigger, as is exhibited in (17) or (18). This is quite different from the previous results gained from the similar exclusive decay processes[13, 17], where the branching ratios go up as d_U is increasing. It is also important to note the different definitions of the couplings from Ref.[17].

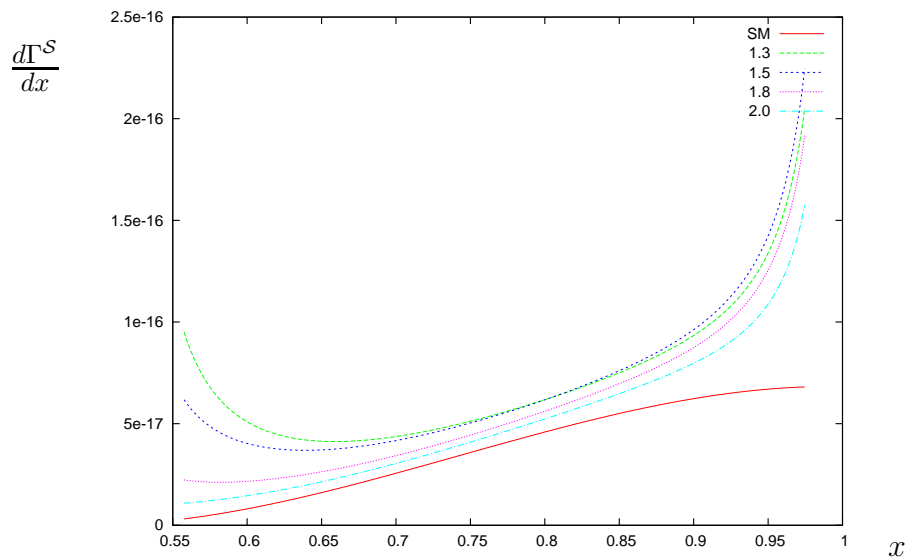


Figure 1: The scalar unparticle energy spectrum in inclusive $B \rightarrow X_s + \cancel{E}$ with $c_S^{q^2} = 1 \times 10^{-17}$. $d_U = 1.3, 1.5, 1.8$ and 2 , increasing from top to bottom. The solid line represents the pure SM result. We cut at $x = 0.975$.

5 Summary

In this paper we have discussed the process $B \rightarrow X_s + \text{Missing Energy}$ in the unparticle model and given some analytical results of the decay withes in free quark limit and the differential decay rates to the $1/m_b^2$ order. If $d_U = 1.3 \sim 1.4$, the unparticle stuffs is most likely to be tested. If one regards the conformal symmetry[6], the vector unparticle has the most distinctive effect around $d_U = 3$. Near the lower endpoint region $x \sim (m_B^2 - m_K^2)/2m_B m_b$ in the spectrum, the unparticle model show very distinctive behavior from the SM. This is very possible to be tested in experiments.

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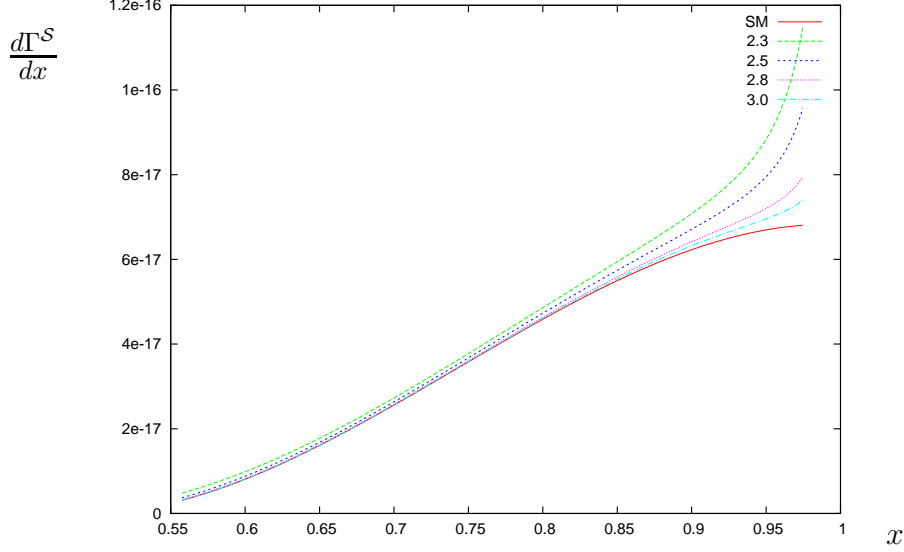


Figure 2: The scalar unparticle energy spectrum in inclusive $B \rightarrow X_s + \cancel{E}$ with $c_S^{q^2} = 1 \times 10^{-17}$. $d_U = 2.3, 2.5, 2.8$ and 3 , increasing from top to bottom. The solid line represents the pure SM result. The cut is at $x = 0.975$.

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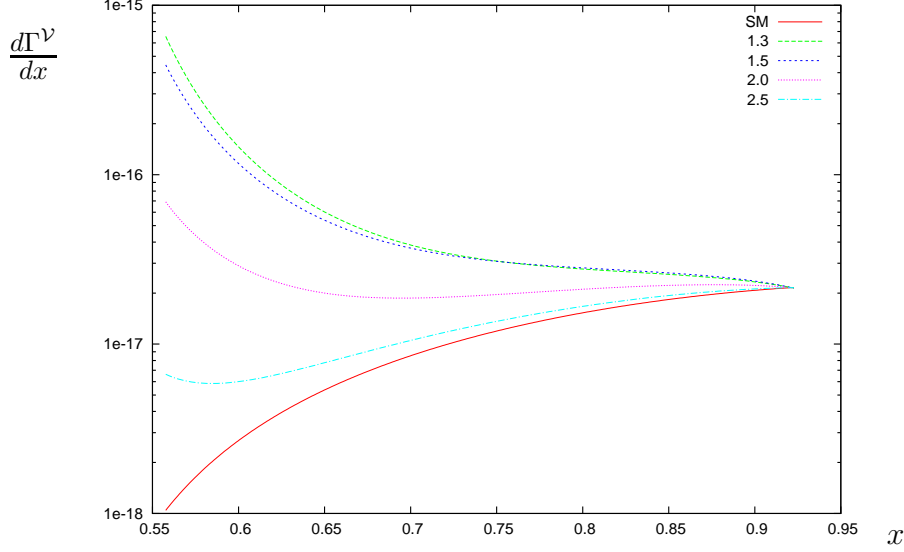


Figure 3: The vector unparticle energy spectrum in inclusive $B \rightarrow X_s + \cancel{E}$ with $c_V^{q^2} = 1 \times 10^{-15}$. $d_U = 1.3, 1.5, 2.0$ and 2.5 , increasing from top to bottom. The solid line represents the pure SM result. The cut is at $x = 0.923$.

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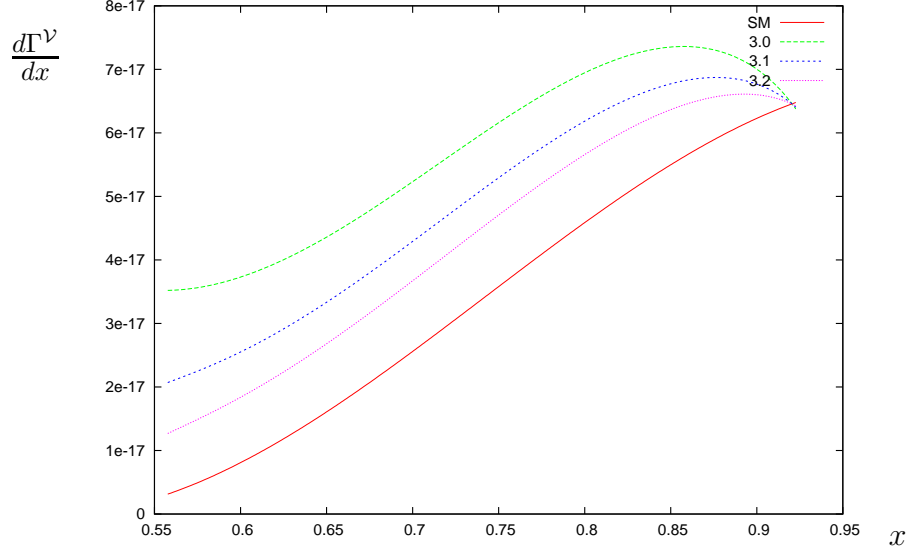


Figure 4: The vector unparticle energy spectrum in inclusive $B \rightarrow X_s + \cancel{E}$ with $c_V^{q^2} = 1 \times 10^{-14}$. $d_U = 3.0, 3.1$, and 3.2 , increasing from top to bottom. The solid line represents the pure SM result. The cut is at $x = 0.923$.

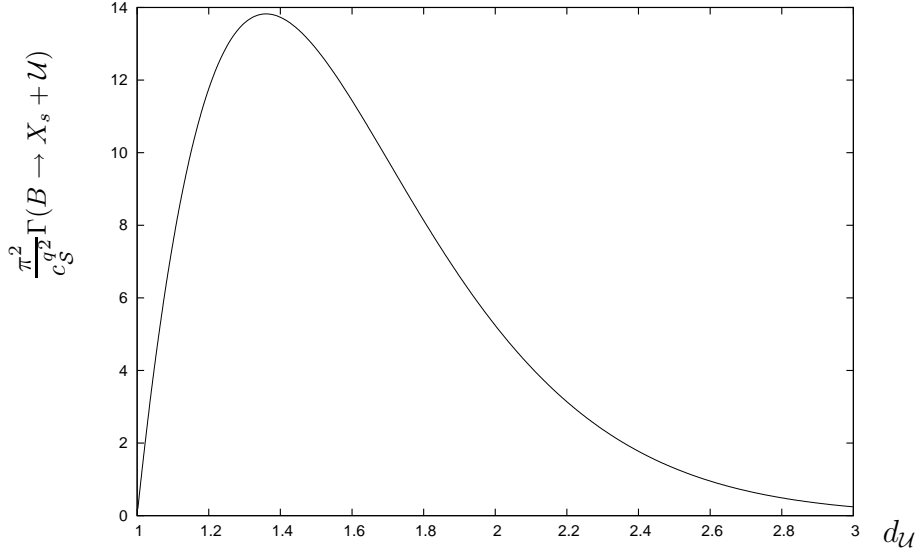


Figure 5: The decay width versus d_U in the scalar unparticle model. It is modulated by $c_S^{q^2}/\pi^2$.

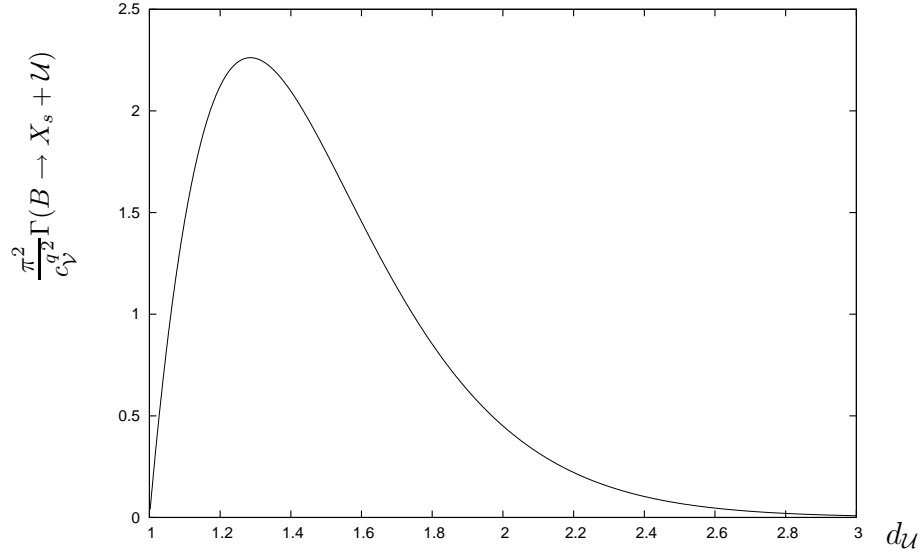


Figure 6: The decay width versus $d_{\mathcal{U}}$ in the vector unparticle model for $d_{\mathcal{U}} < 3$.

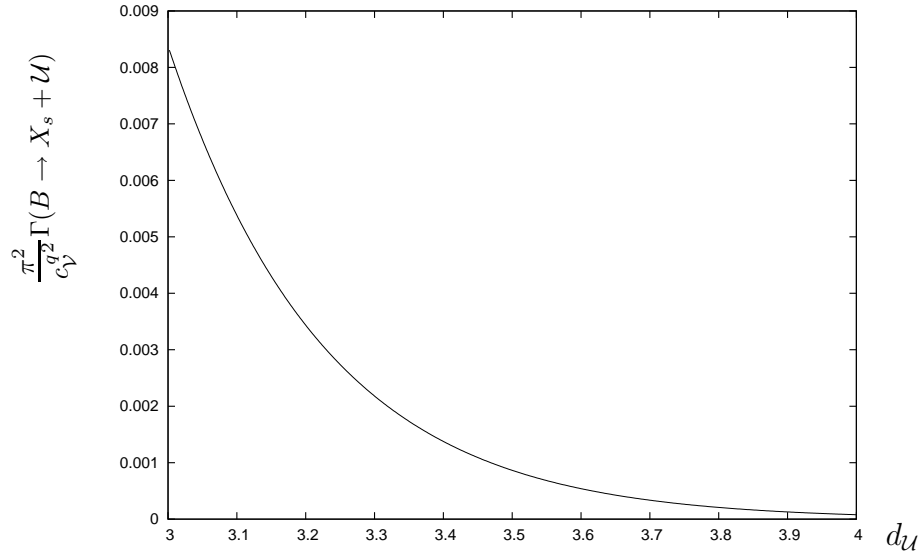


Figure 7: The decay width versus $d_{\mathcal{U}}$ in the vector unparticle model for $d_{\mathcal{U}} \geq 3$.